International Journal of Theoretical Physics, Vol. 29, No. 9, 1990

Radiating Viscous Universes Coupled with Zero-Mass Scalar Field: Exact Solutions

Koijam Manihar Singh[1] and K. S. Bhamra[1]

Received April 4, 1989; revised May 1, 1989

The dynamics of a radiating viscous fluid universe coupled with zero-mass scalar field is investigated in the Einstein formalism and two exact solutions are obtained. Both the solutions give expanding models. Their many physical and geometrical properties are studied. The model universe corresponding to the first solution turns out to be a "big bang" model. The second model shows an interesting feature of absorbing radiation rather than emitting it under certain conditions.

1. INTRODUCTION

It is well known that no real astrophysical object is composed of a perfect fluid. On the other hand, objects with large energy output, either in the form of photons or neutrinos or both, in some phases of their evolution are very much known to exist. A nonstatic distribution would be radiating energy and so it would be surrounded by an ever-expanding zone of radiation. The early universe was an undifferentiated soup of matter and radiation in a state of thermal equilibrium. Gamow (1946) pointed out that in the distant past the universe was dominated by radiation. During the photon decoupling stage, part of electromagnetic radiation behaved as a perfect fluid comoving with matter, while part of it behaved like a unidirectional stream moving with fundamental velocity. Again during the neutrino decoupling stage a similar situation arose in which, apart from streaming neutrinos moving with fundamental velocity, there was a part behaving like a viscous fluid comoving with matter. The discovery of quasistellar objects

[1]Department of Mathematics, Manipur University, Canchipur, Imphal-795003, Manipur, India.

0020-7748/90/0900-1015$06.00/0

and their huge energy requirements motivated various authors to develop a theory of hot convective supermassive stars where general relativistic effects are important. Einstein showed that the linearized equations of gravitational theory revealed the existence of gravitational radiation. Thereafter investigations on the nature and characteristics of radiation and radiation field was carried out by Eddington (1918), Jeans (1926*a*,*b*), Milne (1929), and Thomas (1930). Also, many authors (Sachs, 1961; Heller and Klimek, 1975; Bayin, 1979, 1980; Herrera *et al.*, 1980; Cosenza *et al.*, 1981; Szydlowski and Heller, 1983) studied different aspects of radiating matter distribution. Recently many interesting results on the radiating perfect fluid distribution were obtained by Singh (1988). Analytic solutions of the time-dependent field equations with a nontrivial pressure distribution of perfect fluid and radiation were given by Vaidya (1951*a*,*b*).

On the other hand, scalar fields, as they help in explaining the creation of matter in cosmological theories, represent matter fields with spinless quanta and can describe gravitational fields. Here for our investigation we consider in the following a particular scalar field, namely a scalar meson field which is of zero-mass type and characterizes long-range interactions. Meson particles with the charge of electron and masses of the order of magnitude of 200 electron masses are found in cosmic rays. These particles have a good deal to do with the nuclear forces. The scalar meson field is a matter field and is associated with zero-spin chargeless particles like π and κ mesons. The study of such a field in general relativity has been initiated to provide an understanding of the nature of space-time and the gravitational field associated with neutral elementary particles of zero spin. The concept of scalar fields was introduced by Dirac (1938) in trying to explore the idea of Mach's principle, and he thereby obtained a theory in which the gravitational constant is no longer a constant, but is dependent on time. Thereafter Das (1962), Hyde (1963), Penney (1969), Das and Agarwal (1974), Rao *et al.* (1976), and Gurses (1977) investigated physically realistic solutions on the behavior of the scalar fields. Banerjee and Santosh (1981), Froyland (1982), and Accioly *et al.* (1984) discussed and obtained useful solutions for the coupled gravitational and scalar fields. The different aspects and outcomes of the interactions of a viscous fluid distribution with a scalar field were studied by Singh and Bhamra (1987).

However, hardly any work has been done in studying the interaction between a radiating viscous fluid distribution and a zero-mass scalar field. We are thus motivated to take up this problem. We study here the behavior of the scalar field and the radiation field at different stages of the universe. Both fields are found to die away as time passes and also as the radial distance from the center of the model increases. We also study the role of viscosity in both the models obtained here.

2. FIELD EQUATIONS

For this problem we choose the spherically symmetric isotropic line element

$$ds^2 = \exp(\gamma)\, dt^2 - \exp(\beta) \cdot (dr^2 + r^2\, d\theta^2 + r^2 \sin^2 \theta\, d\varphi^2) \tag{1}$$

where β is a function of r and t, and γ is a function of t only.

We consider a frame which, at any point in space, is at rest with respect to the matter located at that point, that is, the frame in which the universe is isotropic and homogeneous, and is so called the comoving frame. Stated differently, the comoving frame is tied to the galactic fluid.

Now the components of fluid velocity in a comoving coordinate system for the metric (1) are given by

$$u^1 = u^2 = u^3 = 0 \qquad \text{and} \qquad u^4 = \exp(-\gamma/2) \tag{2}$$

along with

$$g^{ij} u_i u_j = 1 \tag{3}$$

Here, in our case, we have the energy-momentum tensor $T_{\mu\nu}$ as

$$T_{\mu\nu} = Z_{\mu\nu} + E_{\mu\nu} + S_{\mu\nu} \tag{4}$$

where $Z_{\mu\nu}$ corresponds to the mechanical part of the energy-momentum tensor due to matter and can be taken as the energy-momentum tensor for a viscous fluid, so that

$$Z_{\mu\nu} = \rho u_\mu u_\nu + (p - \zeta\Theta) H_{\mu\nu} - 2\eta\sigma_{\mu\nu} \tag{5}$$

where ρ is the density of the fluid and p the pressure, and η and ζ are the coefficients of shear and bulk viscosity, respectively, which are assumed to be constants for our case. Here Θ is the volume expansion, $H_{\mu\nu}$ is the projection tensor, and $\sigma_{\mu\nu}$ is the shear tensor. Again, $E_{\mu\nu}$ corresponds to the energy-momentum tensor for the spherically symmetric, radially expanding radiation (Vaidya, (1951*a*,*b*) and is given by

$$E_{\mu\nu} = \sigma_1 \omega_\mu \omega_\nu \tag{6}$$

along with

$$\omega^\mu \omega_\mu = 0 \tag{7}$$

and

$$\omega^\mu_{;\nu} \omega^\nu = 0 \tag{8}$$

where σ_1 is the density of the flowing radiation. $S_{\mu\nu}$ corresponds to the energy-momentum tensor for the zero-mass scalar field and is given by

$$S_{\mu\nu} = \varphi_\mu \varphi_\nu - \tfrac{1}{2} g_{\mu\nu} \varphi_k \varphi^k \tag{9}$$

where the scalar potential φ satisfies the Klein–Gordon equation

$$g^{\mu\nu}\varphi_{;\mu\nu}=\varepsilon \tag{10}$$

where ε is the source density of the scalar field. Hence, finally, $T_{\mu\nu}$ can be written as

$$T_{\mu\nu}=\rho u_\mu u_\nu+(p-\zeta\Theta)H_{\mu\nu}-2\eta\sigma_{\mu\nu}+\sigma_1\omega_\mu\omega_\nu+\varphi_\mu\varphi_\nu-\tfrac{1}{2}g_{\mu\nu}\varphi_k\varphi^k \tag{11}$$

Here we note that the orthogonality conditions for viscous fluid are satisfied identically, namely

$$\begin{aligned} H_{\mu\nu}u^\nu&=0\\ \sigma_{\mu\nu}u^\nu&=0\\ \omega_{\mu\nu}u^\nu&=0\\ \dot{u}_\nu u^\nu&=0 \end{aligned} \tag{12}$$

where $\dot{u}_\nu$ is the acceleration and $\omega_{\mu\nu}$ is the rotation tensor.

Now, for the metric (1), the Einstein field equation

$$R_{\mu\nu}-\tfrac{1}{2}Rg_{\mu\nu}=-8\pi G(Z_{\mu\nu}+E_{\mu\nu}+S_{\mu\nu})$$

yields

$$\begin{aligned} &-\left(\frac{\beta'^2}{4}+\frac{\beta'}{4}\right)\exp(-\beta)+\left(\ddot{\beta}+\frac{3}{4}\dot{\beta}^2-\frac{\dot{\beta}\dot{\gamma}}{2}\right)\exp(-\gamma)\\ &=-8\pi G(p-\zeta\Theta)+8\pi G\sigma_1\omega_1\omega^1-4\pi G\\ &\quad\times[\exp(-\beta)\cdot\varphi'^2+\exp(-\gamma)\cdot\dot{\varphi}^2] \end{aligned} \tag{13}$$

$$\begin{aligned} &-\left(\frac{\beta''}{2}+\frac{\beta'}{2r}\right)\exp(-\beta)+\left(\ddot{\beta}+\frac{3}{4}\dot{\beta}^2-\frac{\dot{\beta}\dot{\gamma}}{2}\right)\exp(-\gamma)\\ &=-8\pi G(p-\zeta\Theta)+4\pi G[\exp(-\beta)\cdot\varphi'^2-\exp(-\gamma)\cdot\dot{\varphi}^2] \end{aligned} \tag{14}$$

$$\begin{aligned} &-\left(\beta''+\frac{1}{4}\beta'^2+\frac{2}{r}\beta'\right)\exp(-\beta)+\tfrac{3}{4}\exp(-\gamma)\cdot\dot{\beta}^2\\ &=8\pi G\rho+8\pi G\sigma_1\omega_4\omega^4+4\pi G[\exp(-\beta)\cdot\varphi'^2+\exp(-\gamma)\cdot\dot{\varphi}^2] \end{aligned} \tag{15}$$

$$\begin{aligned} &-\exp(-\gamma)\cdot\dot{\beta}'\\ &=8\pi G\sigma_1\omega_1\omega^4+8\pi G\varphi'\dot{\varphi} \end{aligned} \tag{16}$$

Also from (7) and (8) we get, respectively,

$$\omega^4=\left[\exp\left(\frac{\beta-\gamma}{2}\right)\right]\omega' \tag{17}$$

and

$$\frac{\partial\omega'}{\partial r}+\frac{\partial\omega'}{\partial t}\exp\left(\frac{\beta-\gamma}{2}\right)+\frac{\beta'}{2}\omega^1+\dot{\beta}\omega^1\exp\left(\frac{\beta-\gamma}{2}\right)=0 \tag{18}$$

Again from (10) we have

$$\exp(-\beta)\cdot\varphi''+\left(\frac{2}{r}+\frac{\beta'}{2}\right)\exp(-\beta)\cdot\varphi'+\left(\frac{\dot{\gamma}}{2}-\frac{3}{2}\dot{\beta}\right)$$
$$\times\exp(-\gamma)\cdot\dot{\varphi}-\exp(-\gamma)\cdot\ddot{\varphi}=\varepsilon \tag{19}$$

Now from the above equations we see that the number of unknowns to be determined is greater than the number of equations at hand. Therefore, we try to solve them by assuming some relation between them.

(Note that overdot and prime denote, respectively, partial differentiation with respect to t and r; and a semicolon followed by a subscript denotes covariant differentiation.)

3. SOLUTIONS OF THE FIELD EQUATIONS

From (16) and (17) we get

$$8\pi G\sigma_1\omega_1\omega^1=-8\pi G\varphi'\dot{\varphi}\exp\left(\frac{\gamma-\beta}{2}\right)-\dot{\beta}'\exp\left(\frac{-\gamma-\beta}{2}\right) \tag{20}$$

Again subtracting (14) from (13), we have

$$\left(\frac{\beta''}{2}-\frac{\beta'^2}{4}-\frac{\beta'}{2r}\right)\exp(-\beta)=8\pi G\sigma_1\omega_1\omega^1-8\pi G\exp(-\beta)\cdot\varphi'^2 \tag{21}$$

Now (20) and (21) give

$$\frac{\beta''}{2}-\frac{\beta'^2}{4}-\frac{\beta'}{2r}=-8\pi G\varphi'\dot{\varphi}\exp\left(\frac{\beta+\gamma}{2}\right)-8\pi G\varphi'^2-\dot{\beta}'\exp\left(\frac{\beta-\gamma}{2}\right) \tag{22}$$

Here we assume

$$\beta=f(r)+g(t) \tag{23}$$

and

$$\varphi=\frac{h(r)}{(8\pi G)^{1/2}}+\frac{k(t)}{(8\pi G)^{1/2}} \tag{24}$$

Then (22) becomes

$$\frac{1}{h'}\left(\frac{f''}{2}-\frac{f'^2}{4}-\frac{f'}{2r}+h'^2\right)\exp\left(-\frac{f}{2}\right)=-\dot{k}\exp\left(\frac{g+\gamma}{2}\right) \tag{25}$$

Since the left-hand side is a function of r only, whereas the right-hand side is a function of t, only, we can equate both of them to a constant. Thus, now (25) separates into

$$\dot{k} \exp\left(\frac{g+\gamma}{2}\right) = c \tag{26}$$

and

$$\frac{1}{h'}\left(\frac{f''}{2} - \frac{f'^2}{4} - \frac{f'}{2r} + h'^2\right) \exp\left(-\frac{f}{2}\right) = -c \tag{27}$$

where c is an arbitrary constant.

If we take

$$g(t) = Ak(t) \tag{28}$$

and

$$\gamma(t) = Bk(t) \tag{29}$$

where A and B are arbitrary constants, then from (26) we get

$$\dot{k} = c \exp\{-\tfrac{1}{2}(A+B)k\}$$

which gives

$$k = \frac{2}{A+B} \log\left(\frac{A+B}{2}\right) + \frac{2}{A+B} \log(d+ct)$$

that is,

$$k = -D \log D + D \log(d+ct) \tag{30}$$

where $D = 2/(A+B)$, and d is an arbitrary constant.

3.1. Case I

Now a solution of equation (27) is

$$\begin{aligned} f &= 2b \log(ar) \\ h' &= \frac{1}{2r}\{[c^2a^{2b}r^{2b+2} + 4b(b+2)]^{1/2} - ca^br^{b+1}\} \end{aligned} \tag{31}$$

where a and b are arbitrary constants.

Then (23) and (29), respectively, give

$$\beta = 2b \log(ar) + AD \log(d+ct) - AD \log D \tag{32}$$

$$\gamma = BD \log(d+ct) - BD \log D \tag{33}$$

Also from (24), (30), and (31) we get

$$\varphi = \tfrac{1}{2}(8\pi G)^{-1/2}(b+1)^{-1}([4b(b+2)+c^2a^{2b}r^{2(b+1)}]^{1/2} + 2(b^2+2b)^{1/2}\log\{[4b(b+2)+a^{2b}c^2r^{2(b+1)}]^{1/2}-2[b(b+2)]^{1/2}\} - 2[b(b+2)]^{1/2}\log(ca^br^{b+1}) - ca^br^{b+1}) \tag{34}$$

Again, using (32), we find that equation (18) becomes

$$\frac{\partial\omega^1}{\partial r}+\frac{\partial\omega^1}{\partial t}\exp\left(\frac{\beta-\gamma}{2}\right)+\frac{b}{r}\omega^1+\dot{\beta}\omega^1\exp\left(\frac{\beta-\gamma}{2}\right)=0$$

the solution of which is

$$\omega^1 = zD^{AD}r^{-b}(d+ct)^{-AD} \tag{35}$$

where z is a constant of integration.

Therefore, from (17) we have

$$\omega^4 = za^bD(d+ct)^{-1} \tag{36}$$

Also, from (20) we get

$$8\pi G\sigma_1\omega_1\omega^1 = -8\pi G\varphi'\dot{\varphi}\exp\left(\frac{\gamma-\beta}{2}\right)$$

Thus,

$$\sigma_1 = \frac{c}{16\pi G}z^{-2}a^{-3b}r^{-b}\{[c^2(ar)^{2b}+4b(b+2)r^{-2}]^{1/2}-c(ar)^b\} \tag{37}$$

From (13) and (14) we have

$$16\pi G(p-\zeta\Theta) = 8\pi G\sigma_1\omega_1\omega^1+\left(\frac{\beta''}{2}+\frac{\beta'^2}{4}+\frac{3\beta'}{2r}\right)\exp(-\beta) - 2\left(\ddot{\beta}+\frac{3}{4}\dot{\beta}^2-\frac{\dot{\beta}\dot{\gamma}}{2}\right)\exp(-\gamma)-8\pi G\exp(-\gamma)\dot{\varphi}^2$$

which gives

$$p = \frac{1}{16\pi G}\Big\{24\pi GAc\zeta D^{BD/2+1}(d+ct)^{-BD/2-1} + D^{AD}\left[b(b+2)a^{-2b}r^{-2b-2}+\frac{c}{2}\right](d+ct)^{-AD} + Ac^2(2+BD-A^{-1}D-\tfrac{3}{2}AD)D^{BD+1}(d+ct)^{-BD-2} - \tfrac{1}{2}cD^{AD}(ar)^{-b}[c^2(ar)^{2b}+4b(b+2)r^{-2}]^{1/2}(d+ct)^{-AD}\Big\} \tag{38}$$

From (15) we get

$$\rho = \frac{1}{32\pi G}\{c^2(3A^2-2)D^{BD+2}(d+ct)^{AD-BD-2} + c^2D^{AD} - 4b(b+2)a^{-2b}D^{AD}r^{-2b-2} - cD^{AD}(ar)^{-b}[c^2(ar)^{2b}+4b(b+2)r^{-2}]^{1/2} - 2b(ar)^{-2b}D^{AD}(b+2)r^{-2}\}(d+ct)^{-AD} \tag{39}$$

From (19) we have

$$\varepsilon = (8\pi G)^{-1/2}D^{AD}(ar)^{-2b}(d+ct)^{-AD}\{[c^2(ar)^{2b} + 4b(b+2)r^{-2}]^{-1/2}[2b(b+1)(b+2)r^{-3} + (b+1)c^2a^{2b}r^{2b-1}] - c(b+1)a^br^{b-1}\} + \tfrac{1}{2}(BD-3AD+2)c^2(8\pi G)^{-1/2}D^{BD+1}(d+ct)^{-BD-2} \tag{40}$$

The luminosity L of this model is given by

$$L = -4\pi r^2 \exp(\beta)\cdot\sigma_1\omega^1 = -\frac{cz^{-1}}{4G}a^{-b}r^2\{[c^2(ar)^{2b}+4b(b+2)r^{-2}]^{1/2} - c(ar)^b\} \tag{41}$$

3.2. Case II

Another solution of equation (27) is

$$\begin{aligned} f &= 2b\log(ar) \\ h' &= -\tfrac{1}{2}\{c(ar)^b + [c^2(ar)^{2b}+4b(b+2)r^{-2}]^{1/2}\} \end{aligned} \tag{42}$$

Now, in this case also, proceeding in the same manner as in the previous case, we obtain

$$\omega^1 = zD^{AD}r^{-b}(d+ct)^{-AD} \tag{43}$$

$$\omega^4 = za^bD(d+ct)^{-1} \tag{44}$$

Here, the scalar potential is given by

$$\varphi = \tfrac{1}{2}(8\pi G)^{-1/2}(b+1)^{-1}(2[b(b+2)]^{1/2}\log(ca^br^{b+1}) - ca^br^{b+1} - [4b(b+2)+(a^bcr^{b+1})^2]^{1/2} - 2[b(b+2)]^{1/2}\log\{[4b(b+2)+(a^bcr^{b+1})^2]^{1/2} - 2[b(b+2)]^{1/2}\}) \tag{45}$$

The radiation density is given by

$$\sigma_1 = -\frac{c}{16\pi G} z^{-2} a^{-3b} r^{-b} \{c(ar)^b + [c^2(ar)^{2b} + 4b(b+2)r^{-2}]^{1/2}\} \tag{46}$$

The fluid pressure is given by

$$\begin{aligned} p = \frac{1}{16\pi G} &(\tfrac{1}{2} c D^{AD} (ar)^{-b} \{[c^2(ar)^{2b} + 4b(b+2)r^{-2}]^{1/2} + c(ar)^b\} \\ &\times (d+ct)^{-AD} + 24\pi G A c \zeta D^{BD/2+1} (d+ct)^{-BD/2-1} \\ &+ A c^2 D^{BD+1} (2 + BD - A^{-1} D - \tfrac{3}{2} AD)(d+ct)^{-AD} \\ &+ b(b+2) a^{-2b} D^{AD} r^{-2b-2} (d+ct)^{-AD}) \end{aligned} \tag{47}$$

The fluid density is given by

$$\begin{aligned} \rho = \frac{1}{8\pi G} &\left\{ \frac{c^2}{4} (3A^2 - 2) D^{BD+2} (d+ct)^{AD-BD-2} \right. \\ &+ \tfrac{1}{4} c^2 D^{AD} + \tfrac{1}{4} c (ar)^{-b} D^{AD} [4b(b+2) r^{-2} + c^2 (ar)^{2b}]^{1/2} \\ &\left. - \tfrac{3}{2} b(b+2) a^{-2b} D^{AD} r^{-2b-2} \right\} (d+ct)^{-AD} \end{aligned} \tag{48}$$

The source density of the scalar field is given by

$$\begin{aligned} \varepsilon = {} & \tfrac{1}{2} c^2 (8\pi G)^{-1/2} (BD + 2 - 3AD) D^{BD+1} (d+ct)^{-BD-2} \\ &- (8\pi G)^{-1/2} D^{AD} (ar)^{-2b} (d+ct)^{-AD} \{[2b(b+1)(b+2) r^{-3} \\ &+ (b+1) c^2 a^{2b} r^{2b-1}][c^2 (ar)^{2b} + 4b(b+2) r^{-2}]^{-1/2} \\ &+ c(b+1) a^b r^{b-1}\} \end{aligned} \tag{49}$$

and

$$L = \frac{c}{4G} z^{-1} a^{-b} r^2 \{[c^2(ar)^{2b} + 4b(b+2) r^{-2}]^{1/2} - c(ar)^b\} \tag{50}$$

4. CONCLUSIONS

4.1. Case I

In this case the line element takes the form

$$\begin{aligned} ds^2 = {} & D^{-BD} (d+ct)^{BD} dt^2 - (ar)^{2b} D^{-AD} (d+ct)^{AD} \\ &\times (dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2) \end{aligned} \tag{51}$$

where a, b, c, d, A, B, and D are arbitrary constants. Here the fluid pressure and the fluid density both are found to be decreasing functions of time. Though both of them are also decreasing functions of r, the fluid density is not found to decrease appreciably. At $t=0$ the pressure and the density both have certain values which happen to be decreasing functions of r, and as $t\to\infty$, both of them tend to zero. At $r=0$, that is, at the center of the model, there is a singularity. Here the effect of viscosity is to increase the pressure, but this effect gradually decreases as time passes. Again, if d happens to be zero, then in that case our model universe turns out to be a "big bang" model.

For this distribution to be a realistic one, we must have (i) $p\geq 0$, (ii) $\rho>0$; and (iii) $\rho\geq p$, which respectively give

$$\begin{aligned}&2Aca^{b}(B+2D^{-1}-A^{-1}-\tfrac{3}{2}A)[(d+ct)D^{-1}]^{AD-BD-2}\\&\quad+48\pi GA\zeta a^{b}[(d+ct)D^{-1}]^{AD-1-BD/2}+a^{b}c\\&\quad+\frac{2b}{c}(b+2)a^{-b}r^{-2b-2}\\&\geq r^{-b}[c^{2}(ar)^{2b}+4b(b+2)r^{-2}]^{1/2}\end{aligned}\tag{52}$$

$$\begin{aligned}&c^{2}\{(3A^{2}-2)[(d+ct)D^{-1}]^{AD-BD-2}+1\}\\&\quad-6b(b+2)a^{-2b}r^{-2b-2}-c(ar)^{-b}\\&\quad\times[4b(b+2)r^{-2}+c^{2}(ar)^{2b}]^{1/2}>0\end{aligned}\tag{53}$$

and

$$\begin{aligned}&c^{2}(3A^{2}D-2A-ABD)D^{BD+1}(d+ct)^{-BD-2}\\&\quad\geq 24\pi GAc\zeta D^{BD/2+1}(d+ct)^{-BD/2-1}\\&\quad+4b(b+2)a^{-2b}D^{AD}r^{-2b-2}(d+ct)^{-AD}\end{aligned}\tag{54}$$

Moreover, from the condition $\rho>0$ we can get one more relation, namely $t>-d/c$. Thus, we see that this solution is valid for a certain interval of time period including the early stages of the universe. Again if $b=-2$ we get the life span of the model universe as

$$\begin{aligned}&\frac{1}{c}\left[\frac{c}{24\pi G\zeta}(3AD-2-BD)D^{BD/2}\right]^{1/(BD+1)}-\frac{d}{c}\geq t\\&\geq\frac{D}{c}\left[\frac{c}{24\pi G\zeta DA}\left(\frac{3}{2}A^{2}D+D-ABD-2A\right)\right]^{2/(2+BD)}-\frac{d}{c}\end{aligned}\tag{55}$$

Here the radiation density is a decreasing function of r. The components ω^1 and ω^4 of radiation are decreasing functions of time, whereas ω^1

decreases also with the increase of radial distance. Thus, on the whole we see that the radiation dies away with the passing of time and also as the radial distance increases. If $b=-2$, then the source density of radiation vanishes, and thus there is no radiation in that case. Again, the luminosity happens to be a function of r only, and at $r=0$, that is, at the center of this model, there is no luminosity.

In this model the source density of the scalar field is a decreasing function of both r and t. As $t\to\infty$ the source density vanishes. The scalar potential φ is also a decreasing function of r and it vanishes if $b=-2$. One peculiarity here is that the scalar field vanishes as soon as the radiation field vanishes and conversely. Again, the source density of the scalar field vanishes if $b=-1$ and $BD+2=3AD$. However, in this case both the scalar field and the radiation field exist.

The components σ_{ij} of shear tensor are all zero; the rotation ω also turns out to be zero.

Here the "expansion factor" Θ of the fluid lines is given by

$$\begin{aligned}\Theta &= \tfrac{3}{2}\dot{\beta}\exp\left(-\frac{\gamma}{2}\right)\\ &= \tfrac{3}{2}AcD^{(BD/2)+1}(d+ct)^{-BD/2-1}\end{aligned} \tag{56}$$

If A, B, c, and D are positive quantities, then we see that the expansion factor is positive and in that case our model universe is an expanding one, but the rate of expansion decreases as time passes.

For this model, the spectral shift will be

$$\frac{\lambda+\delta\lambda}{\lambda}=\left(\frac{d+ct_0}{d+ct}\right)^{-BD/2} \tag{57}$$

Again for a light ray propagated radially from r_1 at t_1 reaching r_2 at t_2 we have

$$\int_{t_1}^{t_2}\exp(\gamma/2)\,dt=\int_{r_1}^{r_2}\exp(\beta/2)\,dr$$

which gives

$$\int_{tt_1}^{t_2}(d+ct)^{(B-A)D/2}\,dt=\int_{r_1}^{r_2}D^{(B-A)D/2}(ar)^b\,dr \tag{58}$$

The integral on the right side is finite, and the integral on the left side converges (and has finite value) for $t_2\to 0$. Thus, we have a particle horizon. We see that an observer at rest at any point in this model could theoretically obtain information concerning sufficiently early states of all parts of the universe; however, though, by waiting for an infinite length of time, the observer could not obtain information as to their behavior later than a certain epoch relating to this model.

4.2. Case II

In this case the fluid pressure as well as the fluid density are found to be decreasing functions of r and t both. As seen from their expressions, both the pressure and the density behave nicely at the beginning of the epoch and as $t \to \infty$ both of them almost vanish. Thus, we see that in the early stages of the evolution of the universe this solution is valid. Again here the effect of viscosity is to increase the pressure, but this effect gradually decreases as time passes; thus, in this model the role of viscosity is more important in the early stages of the evolution of the universe.

Here also the temporal history of this model does not span the entire time period $0 \leq t \leq \infty$, since it is restricted by the reality conditions involving the density and the pressure and we get the limits in this case as

$$\frac{d}{c}-\frac{1}{c}\left[\frac{c}{24\pi G\zeta}(BD+2-3AD)D^{BD/2}\right]^{1/(BD+1)} \geq t$$
$$\geq \frac{d}{c}-\frac{D}{c}\left[\frac{c}{24\pi GA\zeta D}\left(ABD+2A-\frac{3}{2}A^2D-D\right)\right]^{2/(BD+2)} \tag{59}$$

The source density of the scalar field is a decreasing function of time, and as $t \to \infty$ the source density vanishes. The scalar potential φ is a decreasing function of r. In this case also the scalar field vanishes if the radiation field vanishes and vice versa.

Here the "expansion factor" Θ turns out to be

$$\Theta = \tfrac{3}{2}AcD^{BD/2+1}(d+ct)^{-BD/2-1} \tag{60}$$

showing that this model is also an expanding one as in the previous case.

The shear and the rotation are found to be zero in this case.

For this model also there is particle horizon. The spectral shift turns out to be the same as in the previous case,

$$\frac{\lambda+\delta\lambda}{\lambda}=\left(\frac{d+ct_1}{d+ct_2}\right)^{-BD/2} \tag{61}$$

Here the component ω^1 of radiation is found to be a decreasing function of time and radial distance both, but ω^4 happens to be a function of time only and decreases with time. Here the radiation density turns out to be a negative quantity; thus, in this case the distribution would not be radiating energy; rather, it would be absorbing energy. This type of radiation-absorbing distribution is being investigated further. However, if we take c to be a negative quantity, then the radiation density is a positive quantity and it is a decreasing function of r. Here the luminosity is a function of r only. At the center of this model there is no luminosity, but it increases gradually with the radial distance.

REFERENCES

Accioly, A. J., Vaidya, A. N., and Som, M. M. (1984). *Nuovo Cimento B*, **81**, 235.
Banerjee, A., and Santosh, N. O. (1981). *Journal of Mathematical Physics*, **22**, 1075.
Bayin, S. S. (1979). *Physical Review D*, **19**, 2838.
Bayin, S. S. (1980). *Physical Review D*, **21**, 2433.
Cosenza, M., Herrera, L., Esculpi, M., and Witten, L. (1981). *Journal of Mathematical Physics*, **22**, 118.
Das, A. (1962). *Proceedings of the Royal Society A*, **267**, 1.
Das, A., and Agarwal, P. (1974). *General Relativity and Gravitation*, **5**, 359.
Dirac, P. A. M. (1938). *Proceedings of the Royal Society A*, **167**, 148.
Eddington, A. S. (1918). *Monthly Notices of the Royal Astronomical Society*, **79**, 13.
Ellis, G. F. R. (1971). In *General Relativity and Cosmology*, R. K. Sachs, ed., Academic Press, New York.
Froyland, J. (1982). *Physical Review D*, **25**, 1470.
Gamow, G. (1946). *Physical Review*, **70**, 572.
Gurses, M. (1977). *Physical Review D*, **15**, 2731.
Heller, M., and Klimek, Z. (1975). *Astrophysics and Space Science*, **33**, L37.
Herrera, L., Jimenez, J., and Ruggeri, G. J. (1980). *Physical Review D*, **22**, 2305.
Hyde, J. M. (1963). *Proceedings of the Cambridge Philosophical Society*, **59**, 739.
Jeans, J. H. (1926*a*). *Monthly Notices of the Royal Astronomical Society*, **86**, 328.
Jeans, J. H. (1926*b*). *Monthly Notices of the Royal Astronomical Society*, **86**, 444.
Kramer, D., Stephani, H., Herlt, E., and MacCallum, M. A. H. (1980). In *Exact Solutions of Einstein's Field Equations*, E. Schmutzer, ed., Cambridge University Press, Cambridge.
Milne, E. A. (1929). *Monthly Notices of the Royal Astronomical Society*, **89**, 519.
Penney, R. (1969). *Physical Review*, **182**, 1383.
Rao, J. R., Tiwari, R. N., and Nayak, B. K. (1976). *Australian Journal of Physics*, **29**, 195.
Sachs, R. (1961). *Proceedings of the Royal Society A*, **264**, 309.
Singh, K. M. (1988). *International Journal of Theoretical Physics*, **27**, 345.
Singh, K. M., and Bhamra, K. S. (1987). *International Journal of Theoretical Physics*, **26**, 175.
Szydlowski, M., and Heller, M. (1983). *Acta Physica Polonica B*, **14**, 303.
Thomas, L. H. (1930). *Quarterly Journal of Mathematics*, **1**, 239.
Vaidya, P. C. (1951*a*). *Physical Review*, **83**, 10.
Vaidya, P. C. (1951*b*). *Proceedings of the Indian Academy of Sciences, A*, **33**, 264.